

Mathematics and Democracy: Quantitative Tools for Civic Renewal

Reform Elections Now ■ May 6, 2026

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Breaking the partisan gridlock

MATH OF DEMOCRACY CLASS AT WELLESLEY COLLEGE



Mathematics of Democracy, fa...

Description

Course Info

Notes and Materials

Assignments

Group activities

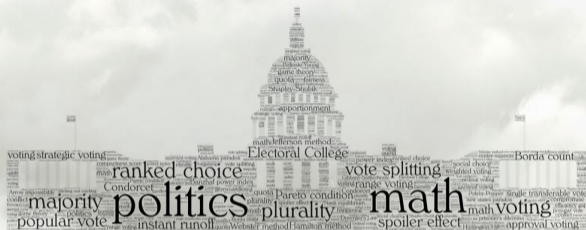
Exam and project

Policies

Grading

Resources

IMD



MATH 123 The Mathematics of Democracy, Fall '25

How can a candidate in a political race win the majority of votes yet lose the election? How can members of a community comprise almost half the electorate yet have no representation in the legislature? How does the shape of a voting district affect whom its inhabitants elect? How much power does the President of the United States have? How can two competing candidates interpret the same statistic as being in their favor?

In this class, we will look at the mathematics behind these and related questions that have bearing on democracy. Some of the topics we will examine are voting methods, apportionment of legislative seats, sizes of legislatures, proportional representation, quantification of power, mathematical tools for detecting gerrymandering, regulation of cryptography and repercussions on privacy, and data interpretation. We will also study the larger context for these topics through an overview of the fields of social choice and game theory which are concerned with the quantification and analysis of notions such as fairness and conflict.



Welcome to

INSTITUTE FOR MATHEMATICS AND DEMOCRACY



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DECLARATION OF INDEPENDENCE IS MATH

- **Axioms:** Self-evident truths.
- **Theorem:** We don't need the British Crown.
- **Proof:** "To prove this, let Facts be submitted to a candid world."



WHY MATH AND DEMOCRACY?

- Voting
- Legislative apportionment
- Sizes of legislatures
- Districting and gerrymandering
- Electoral College
- Proportional representation
- Participatory democracy
- Deliberative democracy
- Digital democracy



WHY MATH AND DEMOCRACY?

- Mathematics can tell us whether these processes are representative and fair.
- It can suggest better ones.
- These topics are uniquely interdisciplinary.
- They can be taught at middle school, high school, college, and graduate level.
- Lots of exciting research currently happening.
- Opportunity to partner with democracy reform organizations.
- The moment calls for it.

Voting

PLURALITY VOTING

- 520,000 officials in the U.S. elected using
 - plurality voting — candidate with the most votes wins.
- Most votes does not mean more than half the votes.
 - vote splitting – similar candidates divide votes, bringing fringe or unpopular candidate to the top;
 - spoiler effect – candidate that has no chance of winning flips an election.
 - Duverger's Law, center-squeeze, strategic voting,...

> MASS. DEMOCRATIC PRIMARY US HOUSE DISTRICT 4

100% REPORTING



Jake Auchincloss

22% | 34,971



Jesse Mermell

21% | 32,938



Becky Grossman

18% | 28,311



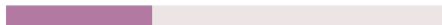
Natalia Linos

12% | 18,158

2021 BOSTON MAYORAL ELECTIONS – RUNOFF DOESN'T HELP



Michelle Wu

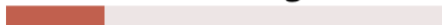


33.4%

35,888 votes



Annissa Essaibi George

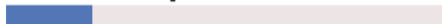


22.5%

24,186 votes



Andrea Campbell

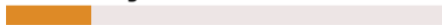


19.7%

21,221 votes



Kim Janey

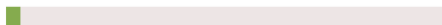


19.5%

20,946 votes



John Barros



3.2%

3,436 votes

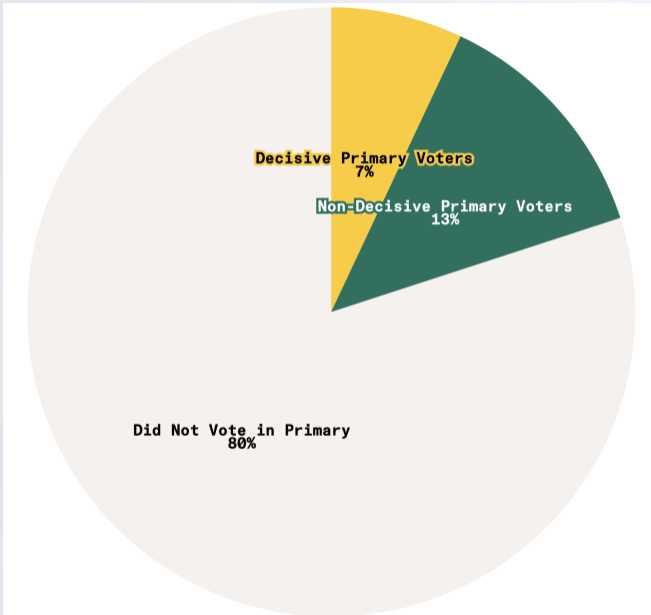
WHY PLURALITY?

- It is the simplest.
- It has been used for a long time (since first presidential elections in 1788).
- People do not know that there are other methods.
- Politicians prefer it.

THE PRIMARY PROBLEM

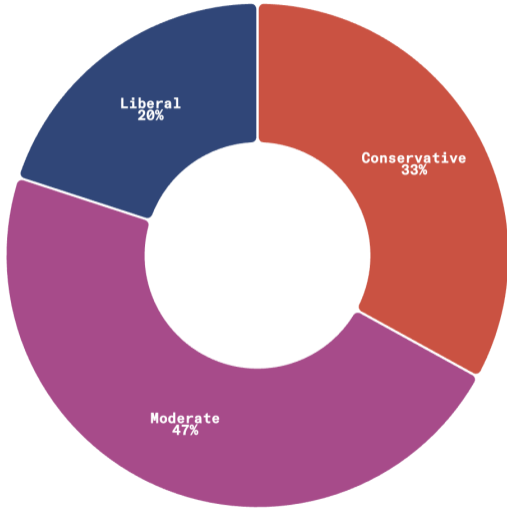
- Plurality problem is evident in the system of primary elections.
- 85% of Congressional races are decided in the primaries.
- In crowded primaries, vote-splitting and spoiler allow a small, dedicated base to elect a candidate.
- Low turnout amplifies the voice of the base.

THE PRIMARY PROBLEM

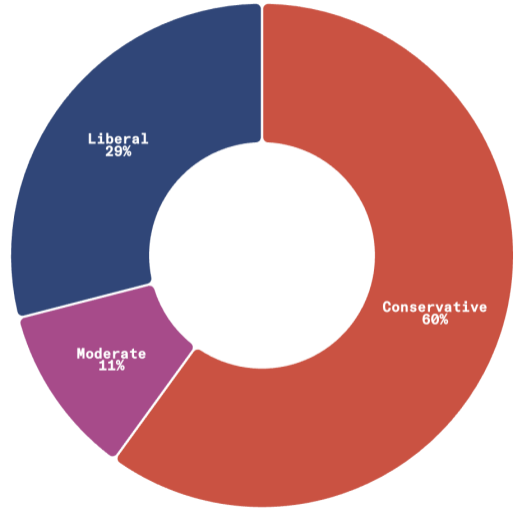


THE PRIMARY PROBLEM

Voting Age Population

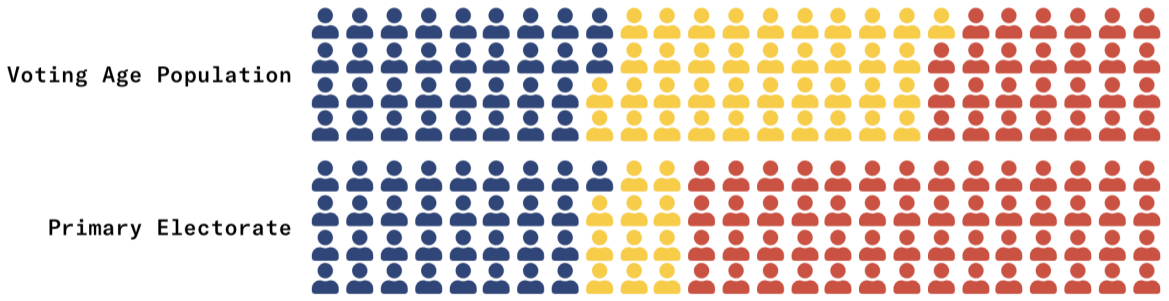


Primary Electorate



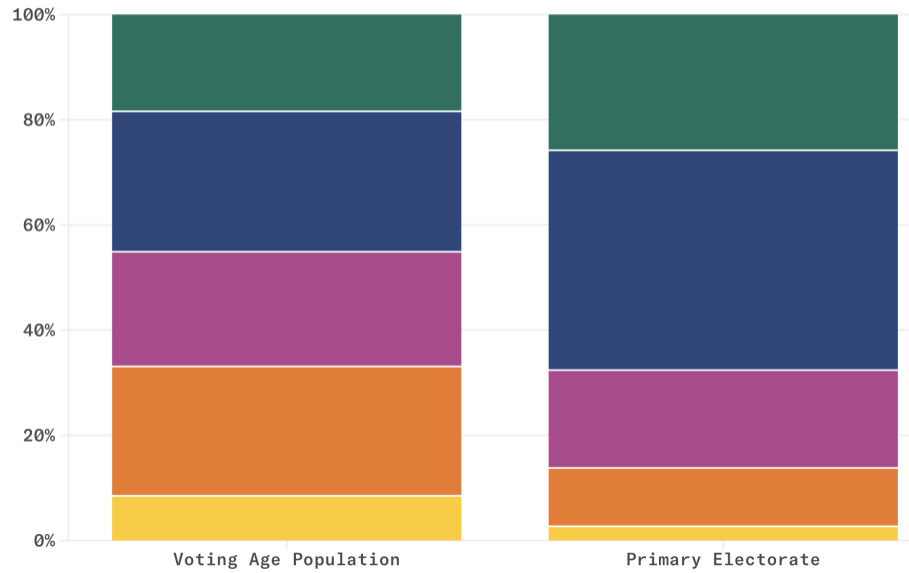
THE PRIMARY PROBLEM

 Democratic  Independent  Republican



THE PRIMARY PROBLEM

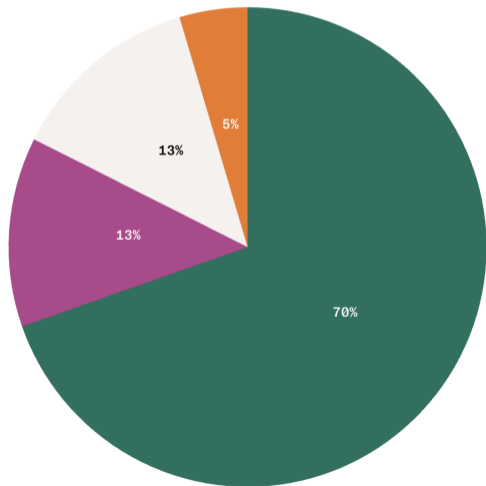
■ <25 ■ 25-39 ■ 40-54 ■ 55-72 ■ 73+



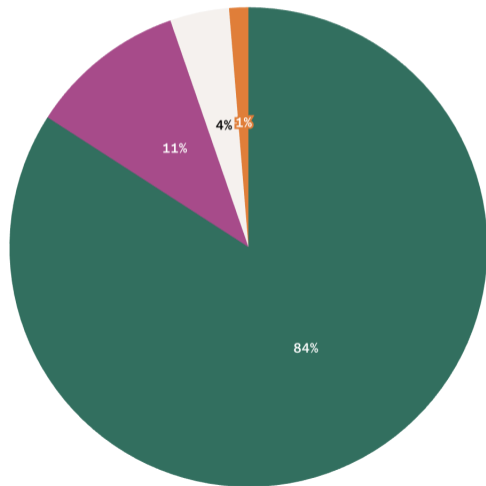
THE PRIMARY PROBLEM

White Black Hispanic Asian

Voting Age Population



Primary Electorate



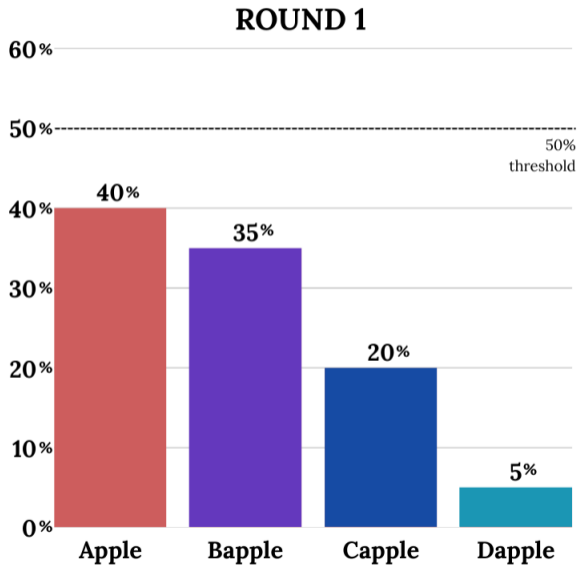
NOW WHAT?

- Plurality takes in very little information.
- Better voting systems ask for more information from the voters.
- One way is to have voters *rank* the candidates.

ranked choice voting

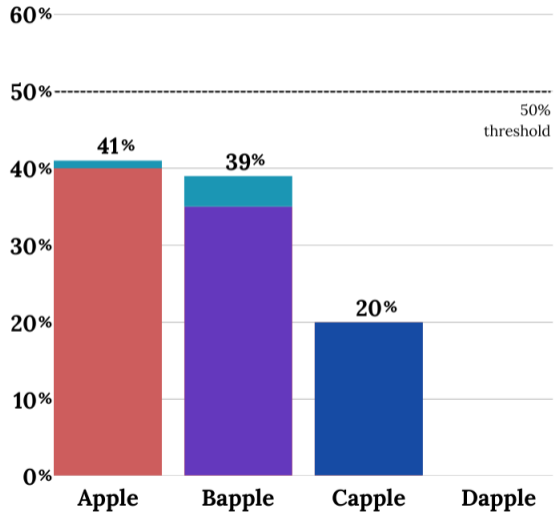
	Preference Ranking			
	1st Choice	2nd Choice	3rd Choice	4th Choice
Candidate A	●	○	○	○
Candidate B	○	○	○	●
Candidate C	○	○	●	○
Candidate D	○	●	○	○

RANKED CHOICE VOTING EXAMPLE

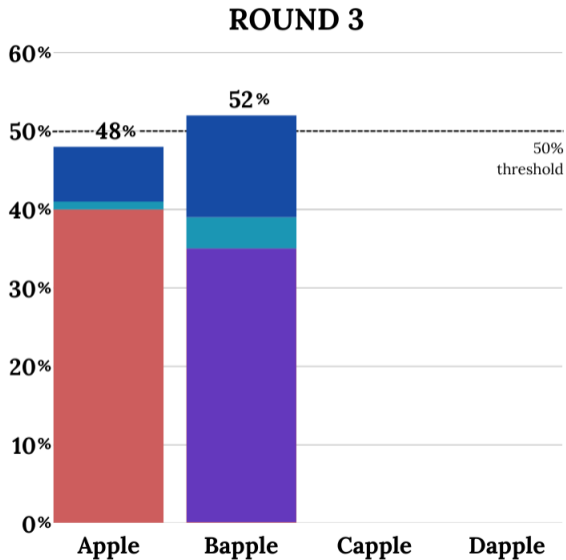


RANKED CHOICE VOTING EXAMPLE

ROUND 2



RANKED CHOICE VOTING EXAMPLE



- Axiomatic framework to study the aggregation of individual preferences into a collective decision, or “the will of the people”.
- How can a group arrive at an outcome that is most beneficial to the individuals?
- When is the decision process democratic and representative?
- How can we rank various alternatives in terms of social well-being?

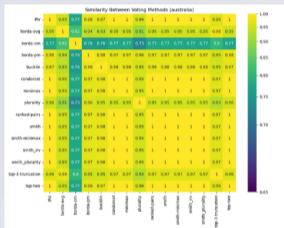
EMPIRICAL RCV METHODS STUDY



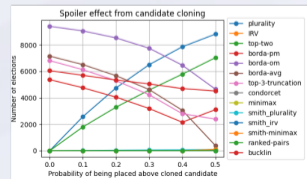
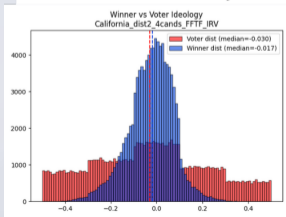
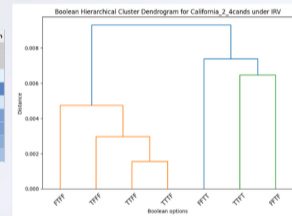
- Adam Graham-Squire (High Point)
- Matthew Jones (Colby College)
- David McCune (William Jewell College)
- Mala Mukundan (Boston University)
- Andy Schultz (Wellesley College)
- Belle See (Wellesley College)
- Ismar Volić (Wellesley College)
- Karen Xiao (Wellesley College)

EMPIRICAL RCV METHODS STUDY

- Analyzed 2,000 ranked elections from the U.S., Scotland, and Australia.
- Generated and analyzed 60 million synthetic elections.
- Ranked choice voting performed very well across many criteria.



	plurality	plurality_runoff	IRV	smith_iv	smith_plurality	minimax	smith_minimax	ranked_pairs	Borda_PM	Borda_OM	Borda_AVG	bucklin
upMono	8	5										
downMono	0	4										
noShow	5		2	0	1							2
laterNoHarm	0	0	2	2	2	2	2	2	571	203	320	248
strat_compromise	133	12	5	2	2	2	2	2	63	115	74	26
strat_truncate_L	0	0	0	3	2	0	0	0	208	0	107	186
strat_truncate_W	0	0	0	3	12	13	14	18	207	0	153	186
strat_bury_shallow	0	0	0	3	15	30	30	34	199	85	166	166
strat_bury_deep	0	0	0	3	32	100	100	108	189	385	295	113



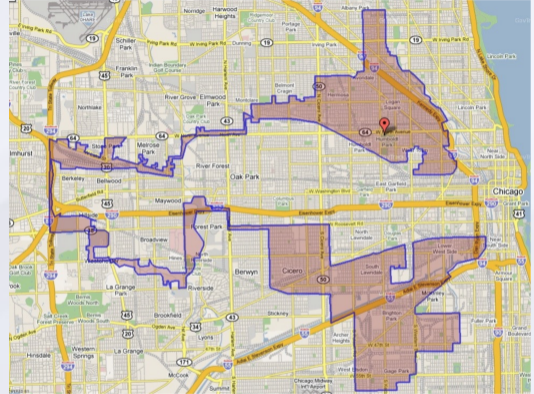


STRANGE DISTRICT SHAPES...



- Many districts have been drawn with political purpose in mind.
- This practice is called **gerrymandering**.
- One of the reasons fewer than 15% of Congressional districts are competitive.

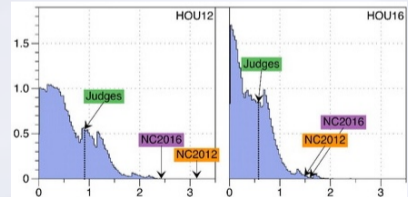
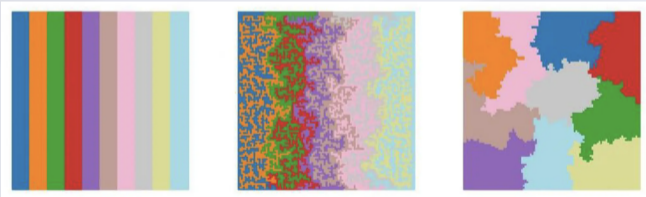
...ARE NOT ALL BAD



Supreme Court recently dismantled Section 2 of the Voting Rights Act.

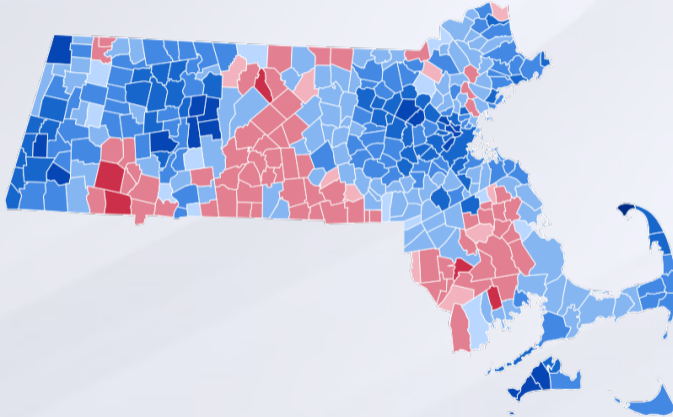
MATHEMATICS OF GERRYMANDERING

- Efficiency gap (wasted votes; packing and cracking)
- Partisan bias (asymmetry in proportionality)
- Compactness scores (geometry of districts)
- Ensemble sampling (comparison to lots of random maps)



THE FIX – PROPORTIONAL REPRESENTATION

- Single-winner plurality + gerrymandering = unrepresentative results.
- 2024 election results in Massachusetts by municipality:

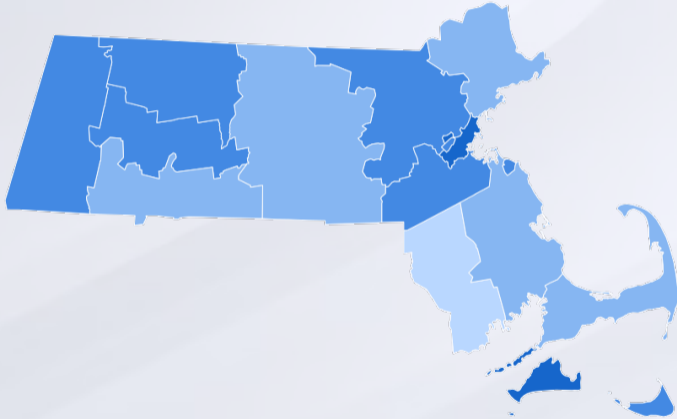


Blue means Democrat won, red means Republican won.

- About 70% voted for Democrats, 30% for Republicans.

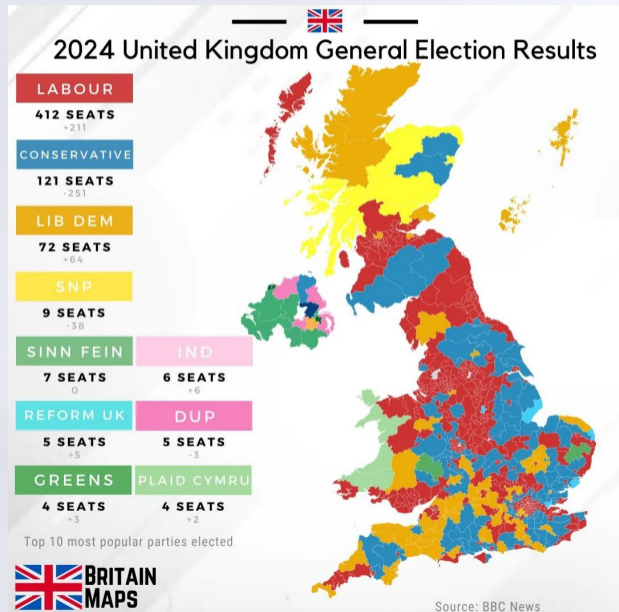
PROPORTIONAL REPRESENTATION

- Massachusetts is divided into 9 districts, each electing one candidate for the U.S. House. Results:



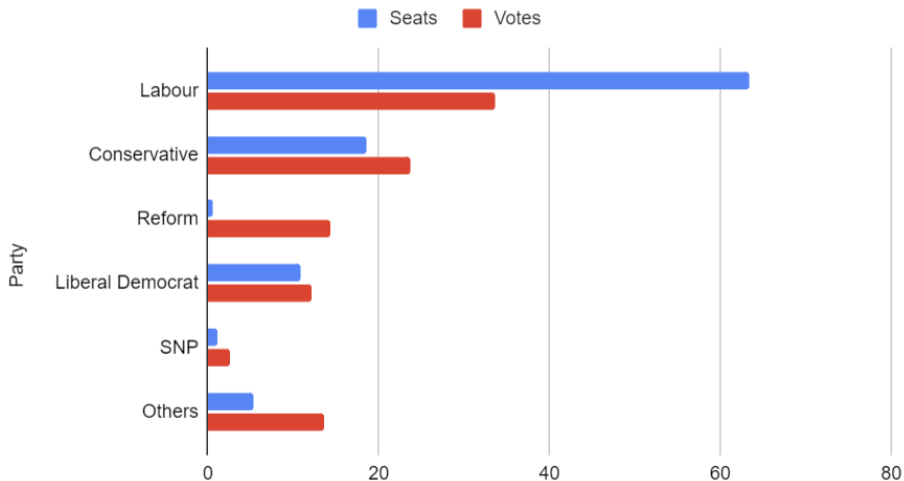
- All nine representatives are Democrats!
- 30% of the voters have no representation.

PROPORTIONAL REPRESENTATION



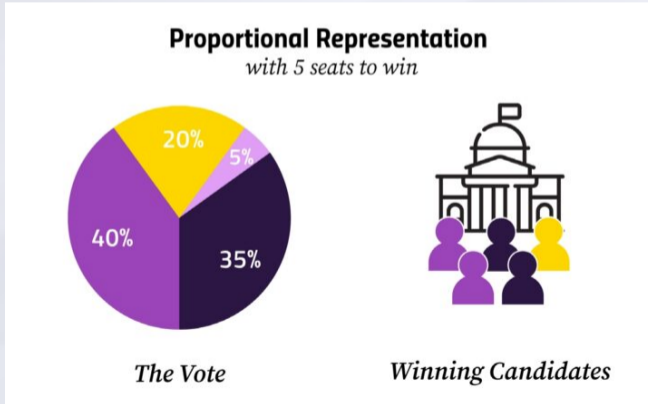
PROPORTIONAL REPRESENTATION

% Seats vs. Votes, UK Election 2024



PROPORTIONAL REPRESENTATION

- A reasonable goal is to achieve
proportional representation (PR).
- PR means % of seats a party gets is (close to) % of votes it gets.



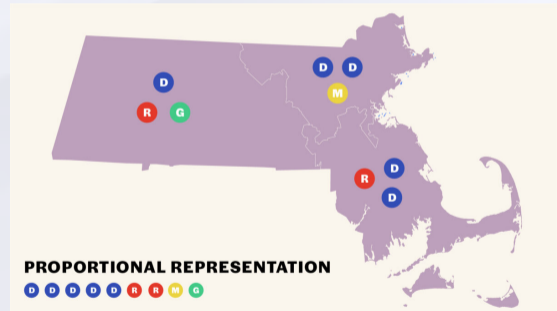
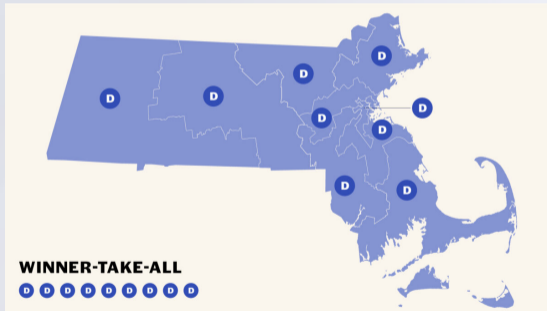
How to get to PR:

- Create larger districts, each electing more than one candidate (3 or 4).
- These are **multi-member districts**.
- Use an election method that achieves proportionality.
- This is what most democracies do.

In the U.S., use *proportional ranked choice voting* to elect multiple representatives in each district.

MASSACHUSETTS SINGLE-WINNER V. PR

- If Massachusetts had 3 districts instead of 9, each electing 3 representatives, and
- if we use a proportional representation system:



PROPORTIONAL REPRESENTATION

- PR essentially eliminates gerrymandering.
- It takes care of the Section 2 of the Voting Rights Act automatically.
- It produces better representation; smaller groups of voters not shut out.
- It encourages participation by smaller parties and candidates.

- Fundamental democratic processes are based on unsound math.
- Using better math can lead to
 - More representative mechanisms and democratic outcomes;
 - Greater civic engagement through mathematical empowerment.
- Math provides a common ground, detached from politics, partisanship, and emotion.

Thank you!